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Induction of Belief Revision Operators from Examples

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Abstract: Propositional belief revision has been widely studied and used to simulate a rational revision of an agent's belief. Although many implementations of belief revision operators have been proposed, they can only be used in an appropriate context. Hence, inductive approach is necessary. We explored the characteristics of a revision operator that can be inducted from examples and proposed simple algorithms that can induce one of the satisfying revision operators.

1 Introduction

Propositional belief revision is a widely used framework to model how agents change their beliefs based on new, trusted information. Rationality of revision operator is standardized through logical constraints[Alchourrón *et al.* 1985 [1], Katsuno and Mendelzon, 1992 [2]]. Many specific revision operators have been proposed based on the mental model of agents and have been proved to satisfy these constraints.[Dalal 1988 [3], Satoh 1988 [4], Katsuno and Mendelzon, 1992 [2]].

Ultimately, the appropriate revision operator should be subject to the context of the problem, such as what propositional logic represents, or what kind of agents revises their own belief. Hence, the data-driven approach is necessary for achieving a realistic revision operator that reflects its context. Let us introduce a motivating example;

Example 1. *You are conducting a questionnaire with a person who knows that there is no atmosphere on Mars but still believes in Martian. You first asked him how would he change his mind if he was told that the atmosphere exists on Mars, but the water does not. He responded that he will still believe in life on Mars. Next, you asked him how would he change his mind if he knows that the atmosphere is necessary for a living to exist, and the questionnaire answered that he would not believe in the Martian in that case. Given these two examples, can we predict how would he react to other new information, such as "water is necessary for life to exist"?*

In section 4, we discussed the induction of revision operator that satisfies one of the most primitive definition of rational revision, *KM postulates.* We analyzed characteristics of revision operators that are consistent with the examples, and propose a method that can induce a single revision operator from examples. In section 5, we first discuss how induction proposed in section 4 does not work well when the number of examples is not enough. We narrowed the domain of revision operator using *distance-based revision operator* to tackle the problem.

2 Preliminary

We assume a finite set of propositional atoms $\mathcal{P}$ and a propositional language $\mathcal{L}_\mathcal{P}$ generated from $\mathcal{P}$ and the usual connectives. An interpretation is a mapping from $\mathcal{P}$ to $\{0,1\}$, and represented as a sequence of atoms that holds true.(e.g. for $\mathcal{P} = \{p,q,r\}$, an interpretation that maps p,q,r to $1,0,1$ respectively is represented as pr.) A set of all interpretations of $\mathcal{L}_\mathcal{P}$ is denoted $\mathcal{W}$. A interpretation ω is a model of $\varphi \in \mathcal{L}_\mathcal{P}$ if and only if it makes it true in the usual truth functional way. $[\varphi]$ denotes a set of models of φ.(i.e. $[\varphi] = \{\omega \in \mathcal{W} \mid \omega \models \varphi\}$). Relation $\models$ represents a logical entailment (i.e. $\varphi \models \psi \iff [\varphi] \subseteq [\psi]$), and $\equiv$ represents a logical equivalence (i.e. $\varphi \equiv \psi \iff [\varphi] = [\psi]$).

A *belief base* is a propositional formula that represents the current belief of agent. A *belief revision operator* $\circ : \mathcal{L}_\mathcal{P} \times \mathcal{L}_\mathcal{P} \to \mathcal{L}_\mathcal{P}$ is a mapping from current belief base K and a new information represented in a formula μ, to a *revised base* $K \circ \mu$.

Example 2 (continued from example 1)**.** *We define a set of propositional atoms $\mathcal{P} = \{p,q,r\}$, where each atom represents a belief as follows:*

- *p: A life exists on Mars.*

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- *q: Atmosphere exists on Mars.*
- *r: Water exists on Mars.*

Then, the 2 responses can be represented as $K_1 \circ \mu_1 = \varphi_1$ *and* $K_2 \circ \mu_2 = \varphi_2$ *respectively using table 2*

i	K_i	μ_i	φ_i
1	$p \wedge \neg q$ (or $\{pr, p\}$)	$p \leftrightarrow q$ (or $\{pqr, pq, r, \emptyset\}$)	$\neg p \wedge \neg q$ (or $\{r, \emptyset\}$)
2	$p \wedge \neg q$ (or $\{pr, p\}$)	$q \wedge \neg r$ (or $\{pq, q\}$)	$p \wedge q \wedge \neg r$ (or $\{pq\}$))

Table 1 Representation of Example 1 using belief revision

Typically, the following postulates are acknowledged as a fundamental core of properties that rational belief revision operator to satisfy.

Definition 1 (KM postulates, Katsuno and Mendelzon, 1992 [2]). *Given* $K, K', \mu, \mu' \in \mathcal{L}_\mathcal{P}$*, KM postulates* (**R1**)-(**R6**) *for belief revision* $\circ$ *is defined as follows:*

(R1) $K \circ \mu \models \mu$
(R2) *If* $K \wedge \mu \not\models \bot$ *then* $K \wedge \mu \equiv K \circ \mu$
(R3) *If* $\mu \not\models \bot$ *then* $K \wedge \mu \not\models \bot$
(R4) *If* $K \equiv K'$ *and* $\mu \equiv \mu'$ *then* $K \circ \mu \equiv K' \circ \mu'$
(R5) $(K \circ \mu) \wedge \mu' \models K \circ (\mu \wedge \mu')$
(R6) *If* $(K \circ \mu) \wedge \mu' \not\models \bot$ *then* $K \circ (\mu \wedge \mu') \models (K \circ \mu) \wedge \mu'$

A revision operator $\circ$ *that satisfies all* (**R1**)-(**R6**) *is called KM revision operator.*

(**R1**) says that the revised belief should incorporate the new information μ. (**R2**) says that the revised belief should be a conjunction of previous belief K and new information μ, whenever they are consistent. (**R3**) says that if μ is consistent, the result of revision should also be consistent. (**R4**) says that the revision should be irrespective of its syntax. (**R5**) together with (**R6**) says that, because $K \circ (\mu \wedge \mu')$ revises K to include μ and φ in a minimal way, if $K \circ \mu$ does not contradict to φ, then the $K \circ (\mu \wedge \mu')$ and $(K \circ \mu) \wedge \mu'$ should be equivalent.

Belief revision operators satisfying all KM postulates can be characterised in terms of total preorders over interpretation, called *faithful assignment*.

Definition 2 (Faithful assignment, Katsuno and Mendelzon, 1992 [2]). *A faithful assignment is a mapping from belief base* $K \in \mathcal{L}_\mathcal{P}$ *to a total preorder* $\leq_K$ *over* $\mathcal{W}$*, such that for any* $K, K' \in \mathcal{L}_\mathcal{P}$*, it satisfies the following conditions:*

1. *If* $\omega \in [K]$ *and* $\omega' \in [K]$*, then* $\omega \simeq_K \omega$*.*
2. *If* $\omega \in [K]$ *and* $\omega' \notin [K]$*, then* $\omega <_K \omega$*.*
3. *If* $K \equiv K'$ *then* $\leq_K = \leq_{K'}$

where $<_K$ *is a strict part of* $\leq_K$*, and* $\simeq_K$ *is a indifference relation of* $\leq_K$*.*

Proposition 1 (Katsuno and Mendelzon, 1992 [2]). *A belief revision operator* $\circ$ *satisfies (**R1**)-(**R6**) if and only if there exists a faithful assignment such that for any* $\mu \in \mathcal{L}_\mathcal{P}$*,* $[K \circ \mu] = \min([\mu], \leq_K)$*.*

In many situations, it is reasonable to assume that the agents choose a set of interpretations from new information that is "close" to the original belief. To follow this idea, *distance-based revision operator* has been proposed.

Definition 3 (Distance-based revision operator, Lehmann et al., 2001 [5]; Konieczny et al., 2004 [6]). *Distance-based revision operator* $\circ_d$ *characterized by pseudo-distance* $d : \mathcal{W} \times \mathcal{W} \to \mathbb{N}_{\geq 0}$ *is a revision operator defined as:*

$$[K \circ_d \mu] = \min([\mu], \leq_K^d)$$

, where for any $\omega_1, \omega_2 \in \mathcal{W}$*,* $\omega_1 \leq_K^d \omega_2$ *if and only if* $d(\omega_1, K) \leq d(\omega_2, K)$*, and for any* $\omega \in \mathcal{W}$*,* $d(\omega, K) = \min_{\omega' \in [K]} d(\omega, \omega')$*.*

It is proven that distance-based revision operators are indeed a family of KM revision operators. The most commonly used distance-based revision operator is Dalal's operator $\circ_{Dal}$ [Dalal 1988 [3]] that uses Hamming distance d_H. We say that revision operator $\circ$ satisfies (**DB**) if and only if there exists a pseudo-distance $d : \mathcal{W} \times \mathcal{W} \to \mathbb{N}$ such that $\circ = \circ_d$.

3 Problem formalization

Sample set of belief revision is a set of 3 tuples $S \subseteq \mathcal{L}_\mathcal{P}^3$, where each element $(K, \mu, \varphi) \in S$ represents a sample such that $K \circ \mu = \varphi$. i-th element of S in a certain order is represented as (K_i, μ_i, φ_i). We say that a belief revision operator $\circ$ *explains* sample set S if and only if for any sample $(K_i, \mu_i, \varphi_i) \in S$, $K_i \circ \mu_i \equiv \varphi_i$ holds. We say that sample set S is *valid* for a condition C if and only if there exists a belief revision operator $\circ$ that explains S and satisfies the condition C.

For any sample set S, we write $S_K = \{(\mu_i, \varphi_i) \mid (K, \mu_i, \varphi_i) \in S\}$. This notion is especially convenient for checking explainable revision operator for a sample set, which is only valid for all KM postulates because only samples in S_K helps to induce the result of revision with initial belief K.

Thanks to proposition 1, the validity of sample set for KM postulates can be checked through the existence of appropriate faithful assignment.

Proposition 2. *Sample set* S *that is valid for KM postulates if and only if there exists a mapping from belief base* $K \in \mathcal{L}_\mathcal{P}$ *to a total preorder* $\leq_K$ *over* $\mathcal{W}$ *such that:*

1. *For any* $(K_i, \mu_i, \varphi_i) \in S$ *and* $\omega, \omega' \in [\varphi_i]$*, it holds* $\omega \simeq_{K_i} \omega'$
2. *For any* $(K_i, \mu_i, \varphi_i) \in S$*,* $\omega \in [\varphi_i]$ *and* $\omega \in [\mu_i \wedge \neg\varphi_i]$*, it holds* $\omega <_{K_i} \omega'$
3. *For any* $K, K' \in \mathcal{L}_\mathcal{P}$*, if* $K \equiv K'$ *then* $\leq_K = \leq_{K'}$

4 Inducing from KM belief revision operators

We will focus on a sample set S that is valid only for all KM postulates. To understand the characteristics of the revision operators that explains S, we shall introduce two revision operators.

Definition 4 ($\circ_S^+$ and $\circ_S^-$)**.** *For any sample set S which is valid for all KM postulates, $\circ_S^+$ and $\circ_S^-$ are revision operators such that, for any $K, \mu \in \mathcal{L}_\mathcal{P}$,*

- $[K \circ_S^+ \mu] = \mathrm{minimal}([\mu], \leq_K^S)$*, and*
- $[K \circ_S^- \mu] = \min([\mu], \leq_K^S)$

where $\leq_K^S$ is a transitive closure of relation $\leq = \{(\omega, \omega') \mid (\varphi_i, \mu_i) \in S_{K_i}, \omega \in [\varphi_i], \omega' \in [\mu_i]\} \cup \{(\omega, \omega') \mid \omega \in [K], \omega' \in \mathcal{W}\}$.

Note that $\leq_K^S$ is a partial order. Hence its minimum and minimal element can differ, and a set of minimum elements can be empty. Then, revision operators that explain S can be characterized by the following proposition.

Proposition 3. *Given a sample set S valid for all KM postulates and a revision operator $\circ$ that explains S, for any $K, \mu \in \mathcal{L}_\mathcal{P}$, it holds $K \circ_S^- \mu \models K \circ \mu \models K \circ_S^+ \mu$.*

$K \circ_S^+ \mu$ represents what has been at least supported by the revised base, and $K \circ_S^- \mu$ represents what has been at most supported.

Proposition 4. *Given a sample set S valid for all KM postulates, only one revision operator explains S if and only if for any $K, \mu \in \mathcal{L}_\mathcal{P}$, it holds $K \circ_S^- \mu \equiv K \circ_S^+ \mu$.*

This proposition tells us that not only $K \circ_S^- \mu \equiv K \circ_S^+ \mu$ indicates that only one revision operator explains S, but also that $K \circ_S^- \mu \not\equiv K \circ_S^+ \mu$ indicates that more than two revision operators, and we need more samples to limit them to a single operator.

Example 3 (continued from example 1)**.** *From table 2, we have $r \simeq_K^S \emptyset <_K^S pq <_K^S p$ and $r \simeq_K^S \emptyset <_K^S pqr$. Given $[\mu] = \{q, \emptyset\}$, we have $[K \circ_S^- \mu] = [K \circ_S^+ \mu] = \{\emptyset\}$, which show that any $\circ$ that explains S satisfy $[K \circ \mu] = \{\emptyset\}$. Given $[\mu'] = \{pqr, pq, q\}$, we have $[K \circ_S^- \mu'] = \emptyset$ and $[K \circ_S^+ \mu'] = \{pqr, pq\}$, which show that any $\circ$ that explains S satisfy $[K \circ \mu] \subseteq \{pqr, pq\}$.*

Proposition 5. *Given a sample set S valid for all KM postulates,*

- *$\circ_S^+$ and $\circ_S^-$ both explains S.*
- *$\circ_S^+$ satisfies **(R1)**-**(R5)**, but normally not **(R6)**.*
- *$\circ_S^-$ satisfies **(R1)**,**(R2)**,**(R4)**,**(R5)**, but normally not **(R3)** and **(R6)**.*

This proposition shows that both $\circ_S^+$ and $\circ_S^-$ has some potentials to be used as inducted revision operator, but cannot be justified in terms of KM paradigm. To show that there exists a method that induces an operator that explains the sample set and satisfy the KM postulates, we will introduce a new operator.

Definition 5 ($\circ_S^{Chain}$)**.** *$\circ_S^{Chain} : \mathcal{L}_\mathcal{P} \times \mathcal{L}_\mathcal{P} \to \mathcal{L}_\mathcal{P}$ is a revision operator that is defined by $[K \circ_S^{Chain} \mu] = \min_{\omega \in [\mu]} Chain_K(\omega)$, where Chain is the following function:*

- *If $\omega \in [K]$ then $Chain_K(\omega) = 0$*
- *If $\omega \notin [K]$ and there exists φ_i such that $\omega \in [\varphi_i]$, $Chain_K(\omega) = c+1$, where c is a maximum size of a chain of $<_K^S$ such that ω is maximum*
- *Otherwise, $Chain_K(\omega) = \infty$*

Proposition 6. *For any sample set S valid for all KM postulates,*

- *$\circ_S^{Chain}$ explains S*
- *$\circ_S^{Chain}$ satisfies all KM postulates.*

5 Inducing distance based revision operators

The problem of restricting revision operators only by KM postulates is that they do not set any constraints between different K. This means that if the number of samples in S_K is small in certain K, it cannot use any information in $S_{K'}$ for any other K', and can only output trivial results. The following observation explains that in the extreme case, when S_K is empty, it is impossible to obtain a characteristic of $K \circ \mu$ for any $\circ$ that explains S.

Observation. *For any sample set S, K such that $S_K = \emptyset$ and $\mu \in \mathcal{L}_\mathcal{P}$, we have $K \circ_S^+ \mu \equiv \top$ and $K \circ_S^- \mu \equiv \bot$.*

To overcome this problem, we will focus on KM revision operators that satisfy (**DB**). Using proposition 2 and a definition of distance-based operator, we can convert a sample set to a set of equations and inequations that distance should satisfy. Using the example, we will show how distance-based operator can induce result with unknown initial belief K.

Example 4 (continued from 1)**.** *Let us assume that there is another sample (K_3, μ_3, φ_3), such that $[K_3] = \{r\}$, $[\mu_3] = \{pr, p\}$ and $[\varphi_3] = \{pr\}$. According to proposition 2, distance d such that $\circ_d$ explains $S = \{(K_1, \mu_1, \varphi_1), (K_3, \mu_3, \varphi_3)\}$ should satisfy,*

$$\min\{d(pr, r), d(p, r)\} = \min\{d(pr, \emptyset), d(p, \emptyset)\}$$
$$\min\{d(pr, r), d(p, r)\} < \min\{d(pr, pqr), d(p, pqr)\}$$
$$\min\{d(pr, r), d(p, r)\} < \min\{d(pr, pq), d(p, pq)\}$$
$$d(r, pr) < d(r, p)$$

Let K' and μ' a formula such that $[K'] = \{pqr, r\}$

and $[\mu'] = \{pr, p\}$. *Then, because we can derive*

$$\min\{d(pr,r), d(pr,pqr)\} < \min\{d(p,pqr), d(p,r)\}$$

from the conditions, any distance-based revision operator $\circ_d$*that explains* S *holds* $[K' \circ_d \mu'] = \{pr\}$.

We should set up a framework that can express these equations and inequations and can be used to simplify the conditions.

Definition 6 (Distance condition graph)**.** *5-tuple* $G = (V, A, B, \mathcal{W}, M)$ *is a distance condition graph if and only if,*

- $V = \{1, 2, \ldots n\}$
- $A, B \subseteq V \times V$ *are relations such that* (V, A) *and* (V, B) *are digraphs.*
- $\mathcal{W}$ *is a set of interpretations.*
- M *is a mapping from* V *to* $2^{\{W \in 2^{\mathcal{W}} \mid |W|=2\}}$.*(*$M(i)$ *is denoted* M_i*).*

Each node $i \in V$ is associated with a set of 2-size set of interpretations M_i, which represents a value of $\min_{\{m_1,m_2\} \in M_i} d(m_1, m_2)$ (we denote the value by $\min M_i$). An edge (i, j) of digraph (V, A) represents a condition $\min M_i \leq \min M_j$, and an edge (i, j) of digraph (V, B) represents a condition $\min M_i < \min M_j$. We say that distance d over $\mathcal{W}$ *explains* a distance condition graph G if and only if it satisfies such conditions, and a set of all distances that explains G is denoted *explain*(G).

Then, proposition 2 can be represented by distance condition graph as follows.

Proposition 7. *Sample set* S *that is valid for KM postulates and* (**DB**) *if and only if there exists a distance* d *over* $\mathcal{W}$ *that explains distance condition graph* $G = (V, A, B, \mathcal{W}, M)$ *such that*

1. *For any* $(K_i, \varphi_i, \mu_i) \in S$ *and* $\omega, \omega' \in [\varphi_i]$*, there exists* $t, u \in V$ *such that* $(t, u) \in A$, $M_t = \{(\omega, v) \mid v \in [K_i]\}$ *and* $M_u = \{(\omega', v') \mid v' \in [K_i]\}$
2. *For any* $(K_i, \varphi_i, \mu_i) \in S$, $\omega \in [\varphi_i]$ *and* $\omega' \in [\mu_i \wedge \neg\varphi_i]$*, there exists* $t, u \in V$ *such that* $(t, u) \in B$, $M_t = \{(\omega, v) \mid v \in [K_i]\}$ *and* $M_u = \{(\omega', v') \mid v' \in [K_i]\}$

We should note that there can be 2 unique distance condition graphs G, G' such that *explain*$(G) =$ *explain*(G') and we say that G, G' is *equivalent.* Hence, our next approach is to mutate the graph to an equivalent, "minimum" graph. However, we have yet to find the definition of the minimum form of a graph. We will present what we have found so far.

Proposition 8. *For any distance condition graph* $G = (V, A, B, \mathcal{W}, M)$*, there exist a equivalent distance condition graph* $G' = (V', A', B', \mathcal{W}, M')$ *such that,*

1. *For any* $t \in V$*, there exists* $u \in V$ *such that* $(t, u) \in A \cup B$ *or* $(u, t) \in A \cup B$.
2. *Both* (V, B) *and* $(V, A \cup B \cup C)$ *is transitive.*
3. *For any* $(t, u) \in A$*, if* $M_t \cap M_u \neq \emptyset$ *then there exists* $v \in V$ *such that* $(t, v), (v, u) \in A \cup C$.
4. *For any* $(t, u) \in B$*, if* $M_t \cap M_u \neq \emptyset$ *then there exists* $v \in V$ *such that* $(t, v), (v, u) \in B$.
5. *For any* $t, u, u' \in V$*, if* $(t, u), (t, u') \in A$*, then either* $(u, u') \in A \cup C$ *or* $(u', u) \in A \cup C$.
6. *For any* $t, u, u' \in V$*, if* $(t, u), (t, u') \in B$*, then either* $(u, u') \in B$ *or* $(u', u) \in B$.
7. *For any* $t, u \in V$*,* $M_t \neq M_u$.
8. $A \cap B = A \cap C = \emptyset$

where $C = \{(t, u) \mid M_t \supseteq M_u\}$.

Proof *(sketch).* We can apply any of the following mutations to the graph $G = (V, A, B, \mathcal{W}, M)$ to create a new graph.

1. Select any $t \in V$ such that t does not occur in A nor B. Then , $G' = (V \setminus \{t\}, A, B, \mathcal{W}, M)$ is equivalent to G.
2. Let (V, B') be a transitive closure of (V, B). Then, G and $G' = (V, A, B', \mathcal{W}, M)$ are equivalent because if there exists a chain $t, i_1, \ldots, i_m, u$ in G, the distance that explains G satisfy $\min M_t \leq \min M_{i_1} \leq \ldots \min M_{i_m} \leq \min M_u$ and hence satisfy $\min M_t \leq \min M_u$.
3. Let (V, A') be transitive closure of $(V, A \cup B \cup C)$. Then G and $G' = (V, A', B, \mathcal{W}, M)$ are equivalent because of similar reason to the first mutation (If $(t, u) \in C$, i.e. $M_t \supseteq M_u$, then it holds $\min M_t \leq \min M_u$).
4. Select any $t, u \in V$ such that $(t, u) \in A$. Let M' be a mapping such that $M'_v = M_u \setminus M_t$ for $v \notin V$ and $M'_{t'} = M_{t'}$ for $t' \in V$. Then, $G' = (V \cup \{v\}, A \cup \{(t, v)\} \setminus \{(t, u)\}, B, \mathcal{W}, M')$. This is because $\min M_t \leq \min M_u \iff \min M_t \leq \min M_u \setminus M_t$.
5. Select any $t, u \in V$ such that $(t, u) \in B$. Let M' be a mapping such that $M'_v = M_t \setminus M_u$ for $v \notin V$ and $M'_{t'} = M_{t'}$ for $t' \in V$. Then, $G' = (V \cup \{v\}, A, B \cup \{(v, u)\} \setminus \{(t, u)\}, \mathcal{W}, M')$. This is because $\min M_t < \min M_u \iff \min M_t \setminus M_u < \min M_u$.
6. Select any $t, u, u' \in V$ such that $(t, u), (t, u') \in A$ and let M' be a mapping such that $M'_v = M_u \cup M_{u'}$ for $v \notin V$ and $M'_{t'} = M_{t'}$ for $t' \in V$. Then $G' = (V \cup \{v\}, A \cup \{(t, v)\} \setminus \{(t, u), (t, u')\}, B, \mathcal{W}, M)$ is equivalent to G, because $\min M_t \leq \min M_u$ and $\min M_t \leq \min M_{u'}$ both holds if and only if $\min M_t \leq \min M'_v$ holds.
7. Select any $t, u, u' \in V$ such that $(t, u), (t, u') \in B$ and let M' be a mapping such that $M'_v = M_u \cup M_{u'}$ for $v \notin V$ and $M'_{t'} = M_{t'}$ for $t' \in V$. Then $G' = (V \cup \{v\}, A, B \cup \{(t, v)\} \setminus \{(t, u), (t, u')\}, \mathcal{W}, M)$ is equivalent to G, similarly to the 6th mutation.
8. Select any $t, u \in V$ such that $M_t = M_u$, and let us define $G' = (V \setminus \{u\}, A_{u \to t}, B_{u \to t}, \mathcal{W}, M)$, where $R_{u \to t}$ for $R \in \{A, B\}$ is a relation such that every edge that contains u are replaced by t. Then, G and G' are equivalent.
9. G and $G' = (V, A \cap \overline{B} \cap \overline{C}, B, \mathcal{W}, M)$ are equivalent because if $(t, u) \in C$ then

$\min M_t \leq \min M_u$, and $(t, u) \in A$ then $\min M_t < \min M_u \Rightarrow \min M_t \leq \min M_u$.

Applying 1st mutation multiple times realizes the 1st condition. Applying 2nd and 3rd mutations realizes the 2nd condition. Applying 4th mutation (respectively 5th mutation) multiple times realizes 3rd condition (respectively 4th condition) if G satisfies 8th condition. Applying 6th mutation (respectively 7th mutation) multiple times realizes 5rd condition (respectively 6th condition) if G satisfies 2nd and 8th condition, because $(v, u), (v, u') \in C$. 8th mutation and 9th mutation realizes 7th and 8th condition respectively. □

We predict that such G' is minimum; there does not exist two different distance condition graphs such that satisfy all conditions in proposition 8 and equivalent to G. Though its minimality is yet to be proven, we have not found a counterexample.

We present a method to induce a specific distance that explains a distance condition graph.

Proposition 9. *Given any distance condition graph $G = (V, A, B, \mathcal{W}, M)$ that satisfy the conditions in 8, let d_{Chain} be a distance such that, for any $\omega, \omega' \in \mathcal{W}$*

- *If $\omega = \omega'$, $d(\omega, \omega') = 0$*
- *If there exists $t \in V$ satisfies $\{\omega, \omega'\} \in M_t$, $d(\omega, \omega') = c + 1$, where c is a maximum length of chain of relation A, such that maximum element is t*
- *Otherwise, $d(\omega, \omega') = \infty$*

Together with proposition 7, 8 and 9, we can now induce a distance d from sample set S that is valid for (**DB**).

6 Related Works

[Hunter, 2019, [7]] considered a similar problem. His samples consist of common initial belief base K, new information μ_i and *instance data* $K \circ \mu_i \models G$ or $K \circ \mu_i \not\models G$, and consisted a decision tree that answers whether $K \circ \mu_i \models G$ is true or not. While his paper focused on constructing a single algorithm using a machine learning technique, our research not only proposed multiple operators that can achieve similar results, but also characteristics of the operator that explains the samples.

Several other researches considered an induction of certain information by observing the result of revised belief. [Schwind *et al.*, 2019 [8]] considered a problem where a set of agents changes their belief using belief revision framework, and their goal is to identify a public announcement μ, where each agent's initial belief K_i, their revision operator $\circ_i$ and their revised base $K_i \circ_i \mu$ were given. Although their research also focused on a case where each revision operator $\circ_i$ is unknown, they did not discuss how we can induce the revision operator itself. This departs from our contribution that focus on inducing the operator.

[Booth and Nittka, 2008 [9]] considered the induction problem of iterated belief revision. His inducted the belief state for each iteration i, given a series of what has been at least believed θ_i, what has not been believed D_i, new information φ_i and a revision function. Although their proposed method of reconstructing *epistemic state* has some similarities with reconstructing faithful assignment in this paper, it departs from our contributions that focus on a case when the revision operator is unknown.

7 Future Works

We have shown that narrowing the domain of revision operators by adding a constraint can be efficient when the number of samples is not enough. There are some other family of revision operators that are more specific than distance-based revision operators, such as weighed Dalal's operator, which is a distance-based revision operator that uses weighted Hamming distance $d_H^\rho = \sum_{p \in \mathcal{P}} \rho(p) \, |\omega(p) - \omega'(p)|$. Another family of revision operator is *topic-decomposable distance-based revision operators* proposed by [Konieczny *et al.*, 2017, [10]], which uses complex distance that is created by aggregating simple distances. Induction of such revision operators can be useful. However, they are left as future work.

A method to discover "minimum" and equivalent distance condition graph has not been discovered. Although our mutation over distance condition graph in proposition 8 was sufficient for proposition 9, we believe that finding minimum distance condition graph is essential to understand the characteristics of revision operator that is generating the samples.

Lastly, we have not yet conducted experiments to find some empirical results. Some of the questions that we are interested in are, how many samples do we need for the inducted revision operator to be reasonably close to the original operator, how well did the induction of distance-based revision operator reduce the number of samples needed compared to that of KM revision operators, and how would increasing number of propositional atoms/size of sample set/size of initial belief affect the time required for the induction?

8 Conclusion

We have considered a problem of inducing a belief revision operator given a set of samples. Our goal was to find a revision operator that is consistent with the sample set and satisfy the constraints that should be satisfied if the agent is making a rational revision. First, we investigated induction for KM revision operators and found out that any KM revision operator $\circ$ that explains the sample set can be characterized by a logical interval $K \circ_S^+ \mu \models K \circ \mu \models K \circ_S^- \mu$. We also showed a simple algorithm that can induce one of the KM revision operators. Second, we discussed how only using KM postulates as constraints are not enough in cases where the number of samples is small.

We showed by an example that this could be solved by assuming that the revision operator is distance-based. We have again presented an algorithm to induct a specific distance-based revision operator.

References

[1] Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: Partial meet contraction and revision functions. *The Journal of Symbolic Logic*, Vol. 50, No. 2, pp. 510–530, 1985.

[2] Hirofumi Katsuno and Alberto O. Mendelzon. Propositional knowledge base revision and minimal change. *Artificial Intelligence*, Vol. 52, No. 3, pp. 263–294, 1991.

[3] Mukesh Dalal. Investigations into a theory of knowledge base revision. In *Proceedings of the Seventh National Conference on Artificial Intelligence (AAAI '88)*, p. 475–479, 1988.

[4] Ken Satoh. Nonmonotonic reasoning by minimal belief revision. In *Proceedings of the International Conference on Fifth Generation Computer Systems*, pp. 455–462, 1988.

[5] Daniel Lehmann, Menachem Magidor, and Karl Schlechta. Distance semantics for belief revision. *J. Symb. Log.*, Vol. 66, No. 1, pp. 295–317, 2001.

[6] Sébastien Konieczny, , Jérôme Lang, and Pierre Marquis. DA^2 merging operators. *Artificial Intelligence*, Vol. 157, No. 1, pp. 49 – 79, 2004.

[7] Aaron Hunter. Learning belief revision operators. In *Advances in Artificial Intelligence*, pp. 239–245, 2018.

[8] Nicolas Schwind, Katsumi Inoue, Sébastien Konieczny, Jean-Marie Lagniez, and Pierre Marquis. What has been said? identifying the change formula in a belief revision scenario. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI-19*, pp. 1865–1871, 2019.

[9] Richard Booth and Alexander Nittka. Reconstructing an agent's epistemic state from observations about its beliefs and non-beliefs. *Journal of Logic and Computation*, Vol. 18, pp. 755–582, 02 2008.

[10] Sébastien Konieczny, Jean-Marie Lagniez, and Pierre Marquis. Boosting distance-based revision using SAT encodings. In Alexandru Baltag, Jeremy Seligman, and Tomoyuki Yamada, editors, *Logic, Rationality, and Interaction - 6th International Workshop, LORI 2017, Sapporo, Japan, September 11-14, 2017, Proceedings*, Vol. 10455 of *Lecture Notes in Computer Science*, pp. 480–496, 2017.