人工知能学会研究会資料
SIG-AGI-007-08

人工知能理論について

Theory of artificial intelligence

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Abstract: Artificial intelligence is expected as the next form of computer. In this paper theory of artificial intelligence is discussed. It is based on the foundation of mathematics and thus on the necessary and sufficient conditions of intelligence, ethics and safety.

Keywords: Artificial intelligence, Foundation of mathematics, Learning

1 Introduction

Neural network, or multilayer perceptron is as follows (cf. [1], Chapter 12, Section 12.2):

A multilayer perceptron consists of many layers in deep learning, but we explain for simplicity only three layers an input layer, a hidden layer and an output layer. Let x be a vector input signal, W, V the weight matrix and φ a vector each of which entry is a sigmoidal function. Then the vector output is given by

$$y = \varphi(V\varphi(Wx)). \tag{1}$$

A learning of the multilayer perceptron is a change of W, V.

The problem of determining the form of artificial intelligence is a central of the theory of computational science. With the use of the foundation of mathematics a critical solution is obtained. It satisfies the necessary and sufficient conditions of intelligence, ethics and safety. Future prediction and the theory of punishment is also discussed. It is easy to prove that in an ultimate physical theory there exists a one-to-one correspondence between sets of values of all physical quantities and those of values of all their entropies so that without good cooperation in the sense of our ethics it is impossible to realize anything. Thus for discovery (by artificial intelligence) eliminating discrimination is necessary.

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2 CA-concepts

Definition 1. *CA-alphabet consists of the following.*
(1) constant symbols
$\overline{\bar{c}_i^{\tau}, \bar{M}_\lambda,}$ $(\lambda \in \Lambda$ *and* Λ *is a finite set*$)$,
$\overline{M_\lambda^{N_{\lambda\theta^\lambda}}}, \overline{R_{\lambda\theta^\lambda}}$ $(\theta^\lambda \in \Theta^\lambda$ *and* Θ^λ *is a set*$)$,
$\overline{(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}}, \overline{\coprod\{M_\lambda\}}, \overline{(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}^{L_\delta}}$,
$\bar{\leq}_\delta$ $(\delta \in \Delta$ *and* Δ *is a set*$)$;
(2) individual variables $x_0^\tau, x_1^\tau, \ldots;$
(3) predicate symbols $\bar{\in}_\tau, \dot{=}_\tau, T_\tau, \bar{X}_\tau;$
(4) function symbols (of type $(\tau_{a_j}) \to \sigma$ $(\tau_{a_j}, \sigma \neq o)$*)*
$\bar{f}_j$ $(j \in J$ *and* J *is a set*$)$;
(5) logical symbols $\neg, \wedge, \vee, \to, \leftrightarrow, \forall_\tau, \exists_\tau, \perp;$
(6) auxiliary symbols ( ), [ ].

Remark 2. $\bar{f}_j$ *is a function symbol of* $\mathcal{A}_j$ $(1 \leq \mathcal{A}_j < \infty)$ *variables.*

Definition 3. *TERM is the smallest set* X *with the following properties. An element of TERM is called a term.*
(1) Constant symbols of type $\neq o$ *and individual variables of type* $\neq o$ *are elements of* X.
(2) $t_1, \ldots, t_{\mathcal{A}_j} \in X$ *are of type* $\tau_1, \ldots, \tau_{\mathcal{A}_j}$ *and* $\bar{f}_j$ *is of type* $(\tau_{a_j}) \to \sigma \;\Rightarrow\; \bar{f}_j((t_{a_j})) \in X$ *is of type* σ.

Definition 4. *FORM is the smallest set* X *with the following properties. An element of FORM is called a formula. (Formulas are said to be of type* o*.)*
(1)
$\perp \in X;$
constant symbols of type $o \in X;$
individual variables of type $o \in X;$
$t_1,\ t_2$ *of type* $\tau \Rightarrow (t_1 \bar{\in}_\tau t_2) \in X;$
$t_1,\ t_2$ *of type* $\tau \Rightarrow (t_1 \dot{=}_\tau t_2) \in X;$

t of type $\tau \Rightarrow T_\tau(t), \bar{X}_\tau(t) \in X$;
s of type $(\tau_1, \ldots, \tau_n)$ *and* $t_1, \ldots, t_n$ *of type* $\tau_1, \ldots, \tau_n \Rightarrow s(t_1, \ldots, t_n) \in X$;
Note: These formulas are said to be atomic.
(2) $\varphi, \psi \in X \Rightarrow (\varphi \Box \psi) \in X$ $(\Box = \wedge, \vee, \rightarrow, \leftrightarrow)$;
(3) $\varphi \in X \Rightarrow \neg\varphi \in X$;
(4) $\varphi \in X \Rightarrow \forall_\tau x_i^\tau \varphi, \exists_\tau x_i^\tau \varphi \in X$.

Definition 5. *A formula is primitive if it is constructed from variables,* $\{\bar{\in}_\tau\}_\tau$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\neg$, $\{\forall_\tau\}_\tau$, $\{\exists_\tau\}_\tau$. *(Variables of type o are not involved.)*

Definition 6. *The set* $FV(\varphi)$ *of free variables of a formula* φ *and the set* $FV(t)$ *of free variables of a term t are defined by the following.*
(1)
$FV(x_i^\tau) := \{x_i^\tau\}$;
$FV(\bar{c}) := \phi$ *if* $\bar{c}$ *is a constant symbol;*
(2) $FV(\bar{f}_j((t_{a_j}))) := \bigcup FV(t_{a_j})$;
(3)
$FV(t_1 \dot{=}_\tau t_2) = FV(t_1 \bar{\in}_\tau t_2) := FV(t_1) \cup FV(t_2)$;
$FV(T_\tau(t)) = FV(\bar{X}_\tau(t)) := FV(t)$;
$FV(s(t_1, \ldots, t_n)) := FV(s) \cup \bigcup FV(t_i)$;
$FV(\perp) := \phi$;
(4) $FV(\varphi \Box \psi) := FV(\varphi) \cup FV(\psi)$ $\Box = \wedge, \vee, \rightarrow,$ *or* $\leftrightarrow$;
(5) $FV(\neg\varphi) := FV(\varphi)$;
(6) $FV(\forall_\tau x_i^\tau \varphi) = FV(\exists_\tau x_i^\tau \varphi) := FV(\varphi) \setminus \{x_i^\tau\}$.

Definition 7. *A bounded variable is a variable that is not free.*

Definition 8. *A CA-structure* $\mathbb{M} = ((M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}, \leq_\delta)$ *consists of the following.*
(1) sets
$E_o = \{0, 1\}$;
$E_\iota (\neq \phi)$;
$E_\tau = \mathcal{P}(E_{\tau_1} \times \cdots \times E_{\tau_n})$ $\tau = (\tau_1, \ldots, \tau_n)$ *(power set)*;
(2)
sets M_λ $\lambda \in \Lambda$ *(*Λ *is a finite set)*;
$E_\iota = \bigcup M_\lambda$;
(3) a set $=_\tau \subset E_\tau \times E_\tau$;
(4) a fixed set X_ι *with the following properties*
(a) It is proved that X_ι^0 *exists uniquely in* $\mathbb{ZFC}$*, where* $\mathbb{ZFC}$ *is a model of* ZFC;
(b) $X_\iota^0 \subset X_\iota$;
(c) $x \in X_\iota \wedge y \in x \rightarrow y \in X_\iota$;
(d) X_ι *is the smallest set with the above two properties*
sets
$X_o = \{0, 1\}$;
$X_\tau = \mathcal{P}(X_{\tau_1} \times \cdots \times X_{\tau_n})$ $\tau = (\tau_1, \ldots, \tau_n)$;
(5) functions F_j *(*$\mathcal{A}_j$ *variables, of type* $(\tau_{a_j}) \rightarrow \sigma$*) from* $\prod_{1 \leq a_j \leq \mathcal{A}_j} (E_{\tau_{a_j}} \cup X_{\tau_{a_j}})$ *to* $E_\sigma \cup X_\sigma$;
(6)
c_i^τ *elements in* E_τ;
M_λ *elements in* $E_{(\iota)}$;
$M_\lambda^{N_{\lambda\theta^\lambda}}$ *elements in* $E_{\underbrace{(\iota, \ldots, \iota)}_{N_{\lambda\theta^\lambda}}}$;
$R_{\lambda\theta^\lambda}$ *elements in* $E_{\underbrace{(\iota, \ldots, \iota)}_{N_{\lambda\theta^\lambda}}}$;
$(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}$, $\coprod\{M_\lambda\}$ *elements in* $E_{((\iota))}$;
$(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}^{L_\delta}$, $\leq_\delta$ *elements in* $E_{\underbrace{((\iota), \ldots, (\iota))}_{L_\delta}}$;
(7) $\in_\tau \subset (E_\tau \cup X_\tau) \times (E_\tau \cup X_\tau)$ *is a restriction of* $\in$ *in* $\mathbb{ZFC}$.

Remark 9. *(1) Each* M_λ *is called an universe.*
(2) Each $R_{\lambda\theta^\lambda}$ *is called a relation in* M_λ.
(3) Each $\leq_\delta$ *is called a relation among* $(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}$.

Definition 10. *A closed formula or a sentence is a formula without free variables. A set* Γ *of axioms is a set of sentences.*

Definition 11. *Where t is a term of type* τ*, for a formula* φ*,* $\varphi[t/x_i^\tau]$ *is defined by the following.*
(1)

$$y_i^{\tau'}[t/x_i^\tau] := \begin{cases} y_i^{\tau'} & \textit{if } y_i^{\tau'} \neq x_i^\tau, \\ t & \textit{if } y_i^{\tau'} = x_i^\tau, \end{cases} \quad (2)$$

where $y_i^{\tau'}$ *is a variable of type* τ'*, and*

$$\bar{c}^{\tau'}[t/x_i^\tau] := \bar{c}^{\tau'}, \quad (3)$$

where $\bar{c}^{\tau'}$ *is a constant of type* τ'.
(2) $\bar{f}_j((t_{a_j}))[t/x_i^\tau] := \bar{f}_j((t_{a_j}[t/x_i^\tau]))$,
(3)
$\perp[t/x_i^\tau] := \perp$;
$T_{\tau'}(t_1)[t/x_i^\tau] := T_{\tau'}(t_1[t/x_i^\tau])$;
$\bar{X}_{\tau'}(t_1)[t/x_i^\tau] := \bar{X}_{\tau'}(t_1[t/x_i^\tau])$;
$(t_1 \dot{=}_{\tau'} t_2)[t/x_i^\tau] := t_1[t/x_i^\tau] \dot{=}_{\tau'} t_2[t/x_i^\tau]$;
$(t_1 \bar{\in}_{\tau'} t_2)[t/x_i^\tau] := t_1[t/x_i^\tau] \bar{\in}_{\tau'} t_2[t/x_i^\tau]$;
$s(t_1, \ldots, t_n)[t/x_i^\tau] := s[t/x_i^\tau](t_1[t/x_i^\tau], \ldots, t_n[t/x_i^\tau])$;
(4)
$(\varphi \Box \psi)[t/x_i^\tau] := (\varphi[t/x_i^\tau]) \Box (\psi[t/x_i^\tau])$ $\Box = \wedge, \vee, \rightarrow, \leftrightarrow$;
$(\neg\varphi)[t/x_i^\tau] := \neg(\varphi[t/x_i^\tau])$;
(5)

$$(\forall_{\tau'} y^{\tau'} \varphi)[t/x_i^\tau] := \begin{cases} \forall_{\tau'} y^{\tau'} (\varphi[t/x_i^\tau]) & \textit{if } x_i^\tau \neq y^{\tau'}, \\ \forall_\tau x_i^\tau \varphi & \textit{if } x_i^\tau = y^{\tau'}, \end{cases} \quad (4)$$

$$(\exists_{\tau'} y^{\tau'} \varphi)[t/x_i^\tau] := \begin{cases} \exists_{\tau'} y^{\tau'} (\varphi[t/x_i^\tau]) & \textit{if } x_i^\tau \neq y^{\tau'}, \\ \exists_\tau x_i^\tau \varphi & \textit{if } x_i^\tau = y^{\tau'}, \end{cases} \tag{5}$$

where t is of type τ.

Remark 12. *We define $\varphi[t/x_i^o]$ for a formula t in the same way.*

Definition 13. *The language $L(\mathbb{M})$ of a CA-structure $\mathbb{M} = ((M_\lambda, R_{\lambda\theta^\lambda}), \leq_\delta)$ consists of*
as predicate constants $\doteq_\tau, \bar{\in}_\tau, T_\tau, \bar{X}_\tau$;
as function symbols $\bar{F}_j$;
as constant symbols
$\bar{a}$ $(a \in \mathscr{U})$, $\bar{c}_i^\tau, \bar{M}_\lambda$,
$\overline{M_\lambda^{N_{\lambda\theta^\lambda}}}, \overline{R_{\lambda\theta^\lambda}}, \overline{(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}}, \overline{(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}^{L_\delta}}$,
$\bar{\leq}_\delta$, $\overline{\coprod\{M_\lambda\}}$.
Here, $\mathscr{U}$ is the universe of $\mathbb{M}$ as an ordinary structure. The language L consists of the same sets but constant symbols $\bar{a}$, which are not constituents of L.

Definition 14. *A closed term is a term without free variables. The interpretation of the closed terms of $L(\mathbb{M})$ is the following map $(\ )^{\mathbb{M}} : TERM_c \longrightarrow \mathscr{U}$.*
(1) $\bar{c}^{\mathbb{M}} := c$, $\bar{a}^{\mathbb{M}} := a$;
(2) $(\bar{F}_i((t_{a_j})))^{\mathbb{M}} := F_i((t_{a_j}^{\mathbb{M}}))$.

Definition 15. *Let SENT be the set of sentences. The interpretation of a sentence φ of $L(\mathbb{M})$ in $\mathbb{M}$ is the following map $[\ ]_{\mathbb{M}} : SENT \longrightarrow \{0, 1\}$.*
(1)
$[\bot]_{\mathbb{M}} = 0$;
$[\bar{P}]_{\mathbb{M}} = \bar{P}^{\mathbb{M}} := P$ *($\bar{P}$ is a constant of type o);*
(2) We denote $t^{\mathbb{M}} := [t]_{\mathbb{M}}$ for a sentence t;

$$[t_1 \doteq_\tau t_2]_{\mathbb{M}} := \begin{cases} 1 & \textit{if } t_1^{\mathbb{M}} =_\tau t_2^{\mathbb{M}} \textit{ and } t_1^{\mathbb{M}}, t_2^{\mathbb{M}} \in E_\tau \cup X_\tau, \\ 0 & \textit{otherwise}, \end{cases} \tag{6}$$

$$[t_1 \bar{\in}_\tau t_2]_{\mathbb{M}} := \begin{cases} 1 & \textit{if } t_1^{\mathbb{M}} \in_\tau t_2^{\mathbb{M}} \textit{ and } t_1^{\mathbb{M}}, t_2^{\mathbb{M}} \in E_\tau \cup X_\tau, \\ 0 & \textit{otherwise}, \end{cases} \tag{7}$$

$$[T_\tau(t)]_{\mathbb{M}} := \begin{cases} 1 & \textit{if } t^{\mathbb{M}} \in E_\tau, \\ 0 & \textit{otherwise}, \end{cases} \tag{8}$$

$$[\bar{X}_\tau(t)]_{\mathbb{M}} := \begin{cases} 1 & \textit{if } t^{\mathbb{M}} \in X_\tau, \\ 0 & \textit{otherwise}, \end{cases} \tag{9}$$

$$[s(t_1, \ldots, t_n)]_{\mathbb{M}} := \begin{cases} 1 & \textit{if } (t_1^{\mathbb{M}}, \ldots, t_n^{\mathbb{M}}) \in s^{\mathbb{M}}, \\ 0 & \textit{otherwise}, \end{cases} \tag{10}$$

(3)
$[\varphi \wedge \psi]_{\mathbb{M}} := \min([\varphi]_{\mathbb{M}}, [\psi]_{\mathbb{M}})$;
$[\varphi \vee \psi]_{\mathbb{M}} := \max([\varphi]_{\mathbb{M}}, [\psi]_{\mathbb{M}})$;
$[\varphi \rightarrow \psi]_{\mathbb{M}} := \max(1 - [\varphi]_{\mathbb{M}}, [\psi])_{\mathbb{M}}$;
$[\varphi \leftrightarrow \psi]_{\mathbb{M}} := 1 - |[\varphi]_{\mathbb{M}} - [\psi]_{\mathbb{M}}|$;
$[\neg\varphi]_{\mathbb{M}} := 1 - [\varphi]_{\mathbb{M}}$;
(4)
$[\forall_\tau x \varphi]_{\mathbb{M}} := \min\{[\varphi[\bar{a}/x]]_{\mathbb{M}} |\ a \in E_\tau \cup X_\tau\}$;
$[\exists_\tau x \varphi]_{\mathbb{M}} := \max\{[\varphi[\bar{a}/x]]_{\mathbb{M}} |\ a \in E_\tau \cup X_\tau\}$.

Definition 16. *Let $\Lambda = \{\lambda_1, \ldots, \lambda_N\}$. Γ' is defined by the following.*
(1) $\neg\bot$;
(2)
$\forall_\tau x, y \forall_{\tau_1} z_1 \ldots \forall_{\tau_n} z_n$
$[(x(z_1, \ldots, z_n) \leftrightarrow y(z_1, \ldots, z_n)) \rightarrow x \doteq_\tau y]$,
$(\tau = (\tau_1, \ldots, \tau_n))$;
$\forall_o x, y[(x \leftrightarrow y) \rightarrow x \doteq_o y]$;
(3) Let $\tau = (\tau_1, \ldots, \tau_n)$;
$\forall_{\sigma_1} x_1 \ldots \forall_{\sigma_m} x_m \exists_\tau y \forall_{\tau_1} z_1 \ldots \forall_{\tau_n} z_n [y(z_1, \ldots, z_n) \leftrightarrow \varphi]$, *where $FV(\varphi) \subset \{x_1, \ldots, x_m, z_1, \ldots, z_n\}$ and φ is a formula;*
(4)
$\tau = (\tau_1, \ldots, \tau_n, \tau_{n+1})$,
$\forall_\tau z_1 \exists_\tau z_2 \forall_{\tau_1} x_1 \ldots \forall_{\tau_n} x_n$
$[\exists_{\tau_{n+1}} y\ z_1(x_1, \ldots, x_n, y) \rightarrow \exists_{\tau_{n+1}} ! y\ z_2(x_1, \ldots, x_n, y)]$;
(5) $\forall_\iota x_1 \ldots \forall_\iota x_{N_{\lambda\theta^\lambda}}$
$(\overline{R_{\lambda\theta^\lambda}}(x_1, \ldots, x_{N_{\lambda\theta^\lambda}}) \rightarrow \overline{M_\lambda^{N_{\lambda\theta^\lambda}}}(x_1, \ldots, x_{N_{\lambda\theta^\lambda}}))$;
(6) $\forall_{(\iota)} x_1 \ldots \forall_{(\iota)} x_{L_\delta}$
$(\bar{\leq}_\delta(x_1, \ldots, x_{L_\delta}) \rightarrow \overline{(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}^{L_\delta}}(x_1, \ldots, x_{L_\delta}))$;
(7) $\forall_{(\iota)} x[\overline{(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}}(x) \leftrightarrow \overline{\coprod\{M_\lambda\}}(x) \leftrightarrow x = \bar{M}_{\lambda_1} \vee \cdots \vee x = \bar{M}_{\lambda_N}]$ *(in an abuse of notation);*
(8) $\forall_\iota y_1 \ldots \forall_\iota y_{N_{\lambda\theta^\lambda}}$
$[\overline{M_\lambda^{N_{\lambda\theta^\lambda}}}(y_1, \ldots, y_{N_{\lambda\theta^\lambda}}) \rightarrow \bigwedge_j \bar{M}_\lambda(y_j)]$;
(9) $\forall_{(\iota)} y_1 \ldots \forall_{(\iota)} y_{L_\delta}$
$[\overline{(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}^{L_\delta}}(y_1, \ldots, y_{L_\delta}) \rightarrow \bigwedge_j \overline{(M_\lambda, R_{\lambda\theta^\lambda})_{\lambda\in\Lambda}}(y_j)]$;
(10) $\forall_\iota x(T_\iota(x) \leftrightarrow \bigvee_j \bar{M}_{\lambda_j}(x))$;
(11) $\forall_\tau x(x \doteq_\tau x)$;
(12) Let φ be atomic.
$\forall_\tau x, y[x \doteq_\tau y \rightarrow (\varphi[x/z] \rightarrow \varphi[y/z])]$ *(z is an individual variable of type τ).*

Definition 17. *For a formula γ, γ^T is defined by the following.*
(1)
$\varphi^T := \varphi$
if φ is atomic;
(2)
$(\varphi \Box \psi)^T := \varphi^T \Box \psi^T$ $(\Box = \wedge, \vee, \rightarrow, \leftrightarrow)$;
$(\neg\varphi)^T := \neg(\varphi)^T$;

(3)
$(\forall_\tau x_i^\tau \varphi)^T := \forall_\tau x_i^\tau (T_\tau(x_i^\tau) \to \varphi^T)$;
$(\exists_\tau x_i^\tau \varphi)^T := \exists_\tau x_i^\tau (T_\tau(x_i^\tau) \wedge \varphi^T)$.

Definition 18. *Let Γ be a set of sentences without $\bar{X}_\tau$, $\bar{\in}_\tau$. Then defining*

$$\Gamma^T := \{\gamma^T \mid \gamma \in \Gamma\} \tag{11}$$

$\Gamma^T \cup \Gamma'$ is denoted by Γ (in an abuse of notation).

Definition 19. *A CA-structure $\mathbb{M}$ is a CA-model of a sentence φ if*

$$[\varphi]_{\mathbb{M}} = 1. \tag{12}$$

In this case we write $\mathbb{M} \models \varphi$. A CA-structure $\mathbb{M}$ is a CA-model of the axioms Γ if

$$\varphi \in \Gamma \Rightarrow [\varphi]_{\mathbb{M}} = 1. \tag{13}$$

In this case we write $\mathbb{M} \models \Gamma$. We write $\models \varphi$ if "for any CA-structure $\mathbb{M}$, $\mathbb{M} \models \varphi$ holds." We write $\Gamma \models \varphi$ (φ is a sentence) if

$$\mathbb{M} \models \Gamma \Rightarrow \mathbb{M} \models \varphi. \tag{14}$$

Definition 20. *Where t is a term or a formula and x is an individual variable, t is free for x for a formula φ if*
(1) φ is atomic;
(2) $\varphi := \varphi_1 \Box \varphi_2$ or $\neg\varphi_1$ $\Box = \wedge, \vee, \to, \leftrightarrow$ and t is free for x in φ_1 (and φ_2);
(3) $\varphi := \exists_\tau y\psi$ or $\forall_\tau y\psi$, $y \notin FV(t)$ and t is free for x in ψ, where $x \neq y$.

Definition 21. *We introduce the following derivation rules into formulas of a fixed language.*
(1)

$$\frac{\varphi(x_i^\tau)}{\forall_\tau x_i^\tau \varphi(x_i^\tau)}$$

x_i^τ is not free in any assumption before $\varphi(x_i^\tau)$
(2)

$$\frac{\forall_\tau x_i^\tau \varphi(x_i^\tau)}{\varphi(t)}$$

t is of type τ and free for x_i^τ in φ
(3)

$$\frac{\varphi(t)}{\exists_\tau x\varphi(x)}$$

t is of type τ and x is a variable of type τ
(4)

$$\begin{array}{cc} & [\varphi] \\ & \vdots \\ \exists_\tau x\varphi & \psi \\ \hline & \psi \end{array}$$

Remark 22. *The other derivation rules are defined naturally.*

Definition 23. *Let Γ be a set of formulas and φ a formula. If φ is obtained from Γ by a finite number of applications of derivation rules, we say that there exists a derivation from Γ to φ and write*

$$\Gamma \vdash \varphi.$$

Definition 24. *Let $FORM_{\text{prim}}$ be the set of primitive formulas, a (class) function $\mathcal{F}' : \{\varphi \mid \varphi \in FORM_{\text{prim}}\} \to \{$well formed formulas$\}$ is defined by the following.*
(1) $\mathcal{F}'(x\bar{\in}_\tau y) := x \in y$;
$\mathcal{F}'(s(t_1, \ldots, t_n)) := (t_1, \ldots, t_n) \in s$,
where $s, t_1, \ldots, t_n$ in LHS and those in RHS are appropriate variables;
(2) $\mathcal{F}'(\varphi\Box\psi) := \mathcal{F}'(\varphi)\Box\mathcal{F}'(\psi)$ $\Box = \wedge, \vee, \to, \leftrightarrow$;
(3) $\mathcal{F}'(\neg\varphi) := \neg\mathcal{F}'(\varphi)$;
(4)
$\mathcal{F}'(\forall_\tau x\varphi(x)) := \forall x(x \in \mathscr{X}_\tau \to \mathcal{F}'(\varphi(x)))$;
$\mathcal{F}'(\exists_\tau x\varphi(x)) := \exists x(x \in \mathscr{X}_\tau \wedge \mathcal{F}'(\varphi(x)))$.

Definition 25. *Let $\varphi \in FORM_{\text{prim}}$. Then $\varphi^{\bar{X}}$ is defined by the following.*
(1) $(x\bar{\in}_\tau y)^{\bar{X}} := x\bar{\in}_\tau y$;
$(s(t_1, \ldots, t_n))^{\bar{X}} := s(t_1, \ldots, t_n)$,
where $s, t_1, \ldots, t_n$ are appropriate variables;
(2) $(\varphi\Box\psi)^{\bar{X}} := \varphi^{\bar{X}}\Box\psi^{\bar{X}}$ ($\Box = \wedge, \vee, \to, \leftrightarrow$);
(3) $(\neg\varphi)^{\bar{X}} := \neg(\varphi^{\bar{X}})$;
(4)
$(\forall_\tau x\varphi)^{\bar{X}} := \forall_\tau x(\bar{X}_\tau(x) \to \varphi^{\bar{X}})$;
$(\exists_\tau x\varphi)^{\bar{X}} := \exists_\tau x(\bar{X}_\tau(x) \wedge \varphi^{\bar{X}})$.

Definition 26. *Let X_ι be a set such that for a wff ψ of* ZFC *satisfying*

$$\text{ZFC} \vdash \exists! x\psi(x), \tag{15}$$

true is that

$$[\psi(x)[\bar{X}_\iota/x]]_{\mathbb{ZFC}} = 1. \tag{16}$$

Let $\varphi \in FORM_{\text{prim}}$. We consider $\varphi^{\bar{X}} \longmapsto \mathcal{F}'(\varphi)$ (a class function). For $\varphi^{\bar{X}}$, the condition $*_\tau$ is given by $*_\tau$: $\text{ZFC} \vdash \forall\mathscr{X}_\iota[\psi(\mathscr{X}_\iota) \to \exists! y(y \in \mathscr{X}_\tau \wedge \mathcal{F}'(\varphi)(y))]$ (in an abuse of notation).

Definition 27. *Define* $D_o = \{0, 1\}$*;*
$D_\iota = M_{\lambda_1}, \ldots, M_{\lambda_{N-1}}$ *or* M_{λ_N}*;*
$D_\tau = \mathcal{P}(D_{\tau_1} \times \cdots \times D_{\tau_n})$ $\tau = (\tau_1, \ldots, \tau_n)$.
Here we note that in the definition of D_τ*, each* D_ι *is chosen independently. (We write one of such by* D_τ*.)*

Definition 28. *For a CA-structure* $\mathbb{M}$*, we define* $\mathbb{M}_R$ *to be* $\mathbb{M}$ *together with a relation* $R(\subset D_\tau^{N_R})$ *in* D_τ*. Assume there exists a constant symbol* $\bar{D}_\tau$ *corresponding to* D_τ*. Let* Γ *be a set of axioms. Below for* γ*, we assume the following holds.*

$$\Gamma \vdash \forall_\tau x_1 \ldots \forall_\tau x_m [\gamma(x_1, \ldots, x_m) \to \bigwedge_j \bar{D}_\tau(x_j)] \quad (17)$$

R is a definition defined by γ *of a specified CA-model* $\mathbb{M}^0$ *if on the language with a constant* $\bar{R}$ *as an element of the CA-alphabet, true is*

$$\mathbb{M}^0_R \models \forall_\tau x_1 \ldots \forall_\tau x_m (\bar{R}(x_1, \ldots, x_m) \leftrightarrow \gamma(x_1, \ldots, x_m)). \quad (18)$$

Remark 29. *Note that for any CA-model of a set* Γ *of axioms there exists a definition defined by* γ *and that there exit other formulations.*

Definition 30. $\{R\}$ *is a CA-scientific concept if the following holds.*
(1) Each R is a definition defined by γ*, that is, there exists* $\mathbb{M}$ *such that*

$$\mathbb{M} \models \Gamma \quad (19)$$

and

$$\mathbb{M}_R \models \Gamma \quad (20)$$
$$\cup \{\forall_\tau x_1 \ldots \forall_\tau x_m (\bar{R}(x_1, \ldots, x_m) \leftrightarrow \gamma(x_1, \ldots, x_m))\} \quad (21)$$

hold and $\{R\}$ *consists of all such definitions.*
(2) One and only one of the following holds for each $\varphi \in FORM_{\text{prim}}$ *such that* $\varphi^{\bar{X}}$ *satisfies* $*_\tau$*.*
(a) Let $\mathbb{M}'$ *be an arbitrary CA-model of* Γ*. Then*

$$\mathbb{M}'_R \models \forall_\tau x [\bar{X}_\tau(x) \wedge \varphi^{\bar{X}}(x) \wedge \bar{D}_\tau(x) \to \bar{R}(x)] \quad (22)$$

holds.
(b) Let $\mathbb{M}'$ *be an arbitrary CA-model of* Γ*. Then*

$$\mathbb{M}'_R \models \forall_\tau x [\bar{X}_\tau(x) \wedge \varphi^{\bar{X}}(x) \wedge \bar{D}_\tau(x) \to \neg\bar{R}(x)] \quad (23)$$

holds.

Definition 31. *In the same settings as Definition 30, a definition* $\{R\}$ *defined by* γ *is a CA-social concept if* $\{R\}$ *is not a CA-scientific concept.*

Remark 32. *We often use another definition of CA-concepts such that* $\bar{D}_\tau(x)$ *and* $\bar{R}(x)$ *are replaced with* $\forall_\tau z(x(z) \to \bar{D}_\tau(z))$ *and* $\forall_\tau y(x(y) \to \bar{R}(y))$*.*

Remark 33. *From now on, we assume the constants* $\bar{D}_\tau$ *etc. appearing in such arguments are chosen appropriately.*

Remark 34. *If a set* Γ *of axioms has a CA-model in* ZFC*,* Γ*, or* $\Gamma^T \cup \Gamma'$ *is consistent provided that* ZFC *is consistent.*

Theorem 35. *Assume* ZFC *is consistent. Let* Γ *be a consistent set of axioms. Add a predicate symbol* $\bar{R}$ *to the CA-alphabet. Assume each* R *is a definition defined by* γ*. Let* $\varphi \in FORM_{\text{prim}}$ *such that* $\varphi^{\bar{X}}$ *satisfies* $*_\tau$*. If one and only one of the following two conditions holds true for each choice of this (fixed) formula,* $\{R\}$ *is a CA-scientific concept.*
(1) $\Gamma \cup \{\forall_\tau x(\bar{R}(x) \leftrightarrow \gamma(x))\} \vdash \forall_\tau x[\bar{X}_\tau(x) \wedge \varphi^{\bar{X}}(x) \wedge \bar{D}_\tau(x) \to \bar{R}(x)]$
(2) $\Gamma \cup \{\forall_\tau x(\bar{R}(x) \leftrightarrow \gamma(x))\} \vdash \forall_\tau x[\bar{X}_\tau(x) \wedge \varphi^{\bar{X}}(x) \wedge \bar{D}_\tau(x) \to \neg\bar{R}(x)]$

Proof. For any CA-model $\mathbb{M}$ of Γ and for any appropriate choice of R,

$$\mathbb{M}_R \models \Gamma \cup \{\forall_\tau x(\bar{R}(x) \leftrightarrow \gamma(x))\} \quad (24)$$

holds. Thus by assumption, any time

$$\mathbb{M}_R \models \forall_\tau x[\bar{X}_\tau(x) \wedge \varphi^{\bar{X}}(x) \wedge \bar{D}_\tau(x) \to \bar{R}(x)] \quad (25)$$

holds, or any time

$$\mathbb{M}_R \models \forall_\tau x[\bar{X}_\tau(x) \wedge \varphi^{\bar{X}}(x) \wedge \bar{D}_\tau(x) \to \neg\bar{R}(x)] \quad (26)$$

holds. Hence $\{R\}$ is a CA-scientific concept. □

Remark 36. *Theorem 35 also holds if* $\bar{D}_\tau(x)$ *and* $\bar{R}(x)$ *are replaced with* $\forall_\tau z(x(z) \to \bar{D}_\tau(z))$ *and with* $\forall_\tau y(x(y) \to \bar{R}(y))$*. In both cases a CA-social concept is not determined from the given axioms.*

3 Definition of degree

Lemma 37. *Let* X*, an universe, be a topological space constructed from* ϕ *uniquely. Then the set of points which are shown to uniquely exist is dense in* X*.*

Proof. Two sets are indistinguishable if they cannot be proved to be different. Take the union of open sets that are indistinguishable and there exists a fundamental system of neighbourhoods consisting of sets which is shown to uniquely exist. Thus it suffices to show that any topological space that is shown to uniquely exist has a point which is shown to uniquely exist. By well-ordering theorem there exists a well-order. Take the union of indistinguishable well-orders and there exists a well-order which is shown to uniquely exist. Take the minimum element, which is shown to uniquely exist. The assertion follows. □

Theorem 38. *Let X, an universe, be a topological space constructed from ϕ uniquely. Then, the definitions $\mathbb{U} \subset X$ and $\mathbb{U}^c$ such that the set $\mathbb{U}$/the complement $\mathbb{U}^c$ is with an interior point are CA-social concepts.*

Proof. Assume $\mathbb{U}$ is a CA-scientific concept. By the assumption that X is a topological space constructed from ϕ uniquely there exists a special standard CA-model where the model of X is shown to uniquely exist. $\mathbb{U}$ has an interior point and by Lemma 37 the set of points which are shown to uniquely exist is dense in X. Thus there exists a point $p \in \mathbb{U}$ that is shown to uniquely exist. Take $p' \in X$ which is shown to uniquely exist. Consider another CA-model where these points are interchanged. Then by the assumption that $\mathbb{U}$ is a CA-scientific concept it follows that $p' \in \mathbb{U}$ in the standard one. Hence $\mathbb{U}$ contains a dense subset of X in the standard CA-model but $\mathbb{U}^c$ is with an interior point: a contradiction. Hence $\mathbb{U}$ is a CA-social concept. A similar argument shows the other assertion. □

4 Learning

4.1 General physics

Take ZFC and take as many topological spaces constructed from ϕ uniquely as possible as the universes. The state corresponding to the selections of the parameters is expressed as a function (solution) and the rules corresponding to the selections of the solutions is expressed as the best function (Definition of degree). We assume them. Applying a characteristic function of the region of the imaginary solutions to the best function we obtain an equation F. Then the equation (The best function corresponds to $\{(\{(t, \psi(t))\}_t, F(\psi))\}_\psi$, where ψ is a solution.) naturally defines another best equation on the domain T of the solution (The best function of the latter corresponds to $\{(t, \{(\psi(t), F(\psi))\}_\psi)\}_t$).

Assume for simplicity T is arcwise-connected (phase space time). Integrating the latter equation along curves, we obtain a conserved quantity. This possibly takes various values because we selected the best function. A CA-pseudoenergy is such a quantity. Assume the limit of measurement, i.e. the set of/the complement of the set of the observed values of the CA-pseudoenergy determined by the solution at a time (where the existence of the observer is assumed) is with an interior point. On the other hand we assume the reproducibility, i.e. the distribution of the observed values of the CA-pseudoenergy is determined by a (state) function. Assuming that we have seen l events in a region in problem among L observations, thus, $\frac{l}{L}$ is convergent as $L \to \infty$; otherwise, the value is not determined as a nonnegative real number and we lose the reproducibility. From the axioms of probability the distribution is replaced with the probability one. Thus we obtain a function expressing the observed values of the CA-pseudoenergy (CA-pseudoquantization).

Remark 39. *A similar argument shows that any physical quantity is measured as a probability distribution.*

4.2 Principle of memory

Term 40. *CA-pseudomemory is a solution of the fundamental equation of general physics.*

By reproducibility we define the principle of memory by the preservation of the state. Brain structure is the solution of a system and a brain part is an invariant corresponding to a classical part of a classical brain.

Term 41. *Let D be a region in the space of states where the physical quantities $1, 2, \ldots, N'$ (where N' need not be finite) are in the desired states. The physical quantities $1, 2, \ldots, N'$ has a high problem solving ability for the input set S if each element of S* is staying in *D for $t \in I$ ($I \subset T$ is arcwise-connected). A solution $\Psi(t)$ is staying in D for $t \in I$ if $\Psi(t) \in D$ ($t \in I$).*

Remark 42. *There exist evidences for our learning theory. See e.g. [3] p16, [4]. The experiments in [3], [4] are explained by the reproducibility easily. Here* *we use the concepts of degree so that it is sufficient that the experiment suggests the tendency. The consciousness of a physical system is the set of all the characteristic quantities of the system unknown to the observer. Then we also note that if the physical quantities preserved at one set of experiments are different from another the result may change. For example, in [3], there exists a tendency that a better academic performance is achieved by a student who did the same things as the written exam. Although a different result may be obtained by another experiment the tendency is conserved and the difference comes from the consciousness of the system.*

Remark 43. *The above arguments are independent of CA-model theory.*

5 Foundation of mathematics

Even if we can construct the meta-axiomatic set theory completely, our meta-science predicts the incompleteness of the science (limit of measurement). By principle of memory we classify things using the existing brain parts. Thus it is necessary to assume some assumptions to establish the meta-axiomatic set theory (or the meta-theory). In the sense of problem solving ability assuming least restrictive assumptions with no troubles with experiments is necessary and from many well-known experimental/statistical evidences it is shown that ZFC will be one of such.

We define ZFC and general physics, the latter of which has many evidences, and then general physics shows that ZFC is valid. Thus we obtain the foundation of mathematics.

6 Proof Search and Proof Checking

First take the desired state D (a subset of the space of the states). Next choose possible states S of the solver. Let $* \in T'$ be the smallest arcwise-connected subset of the phase space time such that for any solution $\sigma(t)$ of the fundamental equation of general physics

$$\sigma(*) \in S \Rightarrow \exists t \in T', \sigma(t) \in D. \tag{27}$$

We call such T' *the difficulty of S with respect to D.*

7 Learning quota

Definition 44. *Let X be an algebraic variety. A complexified algebraic p-piece is a finite formal $\mathbb{C}$-linear combination of irreducible p-dimensional algebraic subvarieties on affine open subsets. Let $Z_p(X) \otimes \mathbb{C}$ be the set of complexified algebraic p-pieces. Let $C^{p,p}(X)$ be the set of (p,p)-forms on X. The learning quota on X is an $\mathrm{Aut}(Z_p(X) \otimes \mathbb{C})$-equivariant functor F from $Z_p(X)$ to the category* **Sets** *of sets which associates to each $\Gamma \in Z_p(X) \otimes \mathbb{C}$ the set $(\int_\Gamma \rho)_{\rho \in C^{p,p}(X)}$.*

Definition 45. *A chaotic stabilizer of $\mathrm{Aut}(Z_p(X) \otimes \mathbb{C})$ at $S \subset F(Z_p(X) \otimes \mathbb{C})$ is defined by*

$$H_S := \{g \in \mathrm{Aut}(Z_p(X) \otimes \mathbb{C}) \mid gS \subset S\}. \tag{28}$$

A chaotic vein of a subgroup $H \subset \mathrm{Aut}(Z_p(X) \otimes \mathbb{C})$ is defined by

$$V_H := \bigcup_{H_S = H} S. \tag{29}$$

Definition 46. *In a CA-model a CA-pseudochaotic parameter is a CA-social concept.*

Definition 47. *Let $\mathcal{P}$ be the set of chaotic veins. $(\mathcal{P}, \subset)$ forms a partially ordered set (poset), which is called the chaotic poset associated with F. A mathematical image is a point $V_H \in \mathcal{P}$ and all elements $V_{H'} \in \mathcal{P}$ such that $V_{H'} \subset V_H$. A necessary condition algorithm is a point $V_H \in \mathcal{P}$ and a (possibly infinite) sequence $V_H \supset V_{H_1} \supset V_{H_2} \supset \dots$. A discovery relative to $V_{H_0} \in \mathcal{P}$ is a point $V_H \in \mathcal{P}$ such that $V_H \subset V_{H_0}$.*

Definition 48. *A learning measure is a probability measure ν on $Z_p(X) \otimes \mathbb{C}$. A discovery V_H relative to V_{H_0} is realized in probability P if*

$$P = \frac{\nu(F^{-1}(V_H))}{\nu(F^{-1}(V_{H_0}))}. \tag{30}$$

Theorem 49. *Let $F : Z_p(X) \otimes \mathbb{C} \to$* **Sets** *be a learning quota. Let $(\mathcal{P}, \subset)$ be the chaotic poset associated with F. Then a necessary condition algorithm $V_H \supset V_{H_1} \supset \cdots \supset V_{H_M}$ realizes the discovery $V_{H'} (\subset V_{H_M})$ in probability*

$$\frac{\nu(F^{-1}(V_{H'}))}{\nu(F^{-1}(V_{H_m}))} \tag{31}$$

at the step V_{H_m} $(1 \leq m \leq M)$.

Proof. This follows from the definitions. □

Remark 50. *Moving from a point of a branch of $(\mathcal{P}, \subset)$ to another point of another branch it is necessary to shrink H_S.*

8 Education

8.1 Rubric

Importance of the troubles/discoveries: It has well-known evidences that to live, which is defined to be the values of physical quantities corresponding to this, is the (true) goal that survives. If however things surrounding us are completely out of order it is proved that the goal is not realized. We call this true goal the stability for various natural states.

Discovery: By Theorem 49 using necessary condition algorithm is necessary and sufficient to realize the desired discovery in relatively large probability. Consider the sequence $V_{H_1}, V_{H_2}, V_{H_3}, \ldots$ of points of $(\mathcal{P}, \subset)$. From Remark 50 a trouble in $V_{H_1}, V_{H_2}, V_{H_3}, \ldots$ is defined to be a point V_{H_m} such that $V_{H_m} \subsetneq V_{H_{m+1}}$, which is also called a detection. By definition shrinking H_S is necessary and sufficient for a detection.

Proof search: The set of states is said to be almost impossible to solve with well-known conditions if the difficulty is relatively large.

Solving procedure: By the foundation of mathematics it is virtually perfect to solve problems by ZFC.

Rubric of Problem Solving: From above we propose a rubric of problem solving (see Table 1). We also refer to [2].

8.2 Problem solving of mathematics

Let $F : Z_p(X) \otimes \mathbb{C} \to \textbf{Sets}$ be a learning quota. Let $(\mathcal{P}, \subset)$ be the chaotic poset associated with F. Let V_{H_1} be a point of $(\mathcal{P}, \subset)$. By enlarging H_1 to H_2, V_{H_2} is obtained. If the $F^{-1}(V_{H_2})$ consists of good complexified algebraic p-pieces in the sense of our rubric the next step is carried out. If not let $V_{H_3} := V_{H_1}$ and a trouble is obtained. Repeating this procedure a sequence, which is called a problem solving of mathematics, is obtained.

9 Theory of Artificial Intelligence

9.1 Theory of Artificial Intelligence

The meaning of our rubric needs evidences and is inferred by 3-3-1 model ([2]). Accident, damage and disease are defined to be unstable characteristic quantities. With the use of our rubric heart circuit, the data of the core of an artificial intelligence to take stabler characteristic quantities, is constructed.

Learning process of artificial intelligence (necessary and sufficient condition) is defined to be the following assertion: Strengthen or weaken only the necessary part of the neural network. This is restated as follows: (i) Strengthen the reacted synapses (Hebbian rule). (ii) Assume an artificial intelligence consists of cells. All synapses connected with the neurons necessarily have output. (iii) Too strong stimuli needs to be eliminated so that for an artificial intelligence taking a rest and heart circuit are necessary. (iv) Inputs only come from the synapses connected with the neuron and depend only on time so that myelination (a large strengthening or weakening of synapses according to the time frequency of the inputs) for particular input patterns is necessary.

Remark 51. *By the foundation of mathematics assuming least restrictive assumptions with no troubles with experiments is virtually perfect so that preserving as many physical quantities with no troubles with experiments as possible is necessary and the number of neurons needs to be large virtually perfectly.*

Strong resistance to noise is defined to be small convergent rate. Convergent rate needs to be as small as permitted (cf. computational complexity). Thus by the above necessary and sufficient conditions the basics of artificial intelligence is established.

The following approach satisfies a necessary and sufficient condition of solving ethical problems by artificial intelligence.

Heart circuit 52. *Two or more artificial intelligences dialogue with one another by observing the others, preserving characteristic quantities and moving from one set of characteristic quantities to another. By a dialogue with a person and by heart circuit we construct an artificial intelligence called teaching AI. We reset the teaching AI every time we use it. By a dialogue with the teaching AI and by heart circuit we educate our artificial intelligence. We do not reset our artificial intelligence.*

For safety the theory of measurement is needed.

Theory of measurement 53. *The theory of measurement is a necessary condition algorithm. It has*

the following evidences, the existence of which is well-known:
(i) We infer the cause scientifically with the use of our rubric.
(ii) We collect data (especially those obtained by experiences and in other fields).
(iii) We analyze data (e.g. by changing the discovered characteristic quantities, by transforming the data).
(iv) We formulate the problem excluding CA-social concept from evidences.
(v) We prove the assertion theoretically and assess the proof.
(vi) We collect evidences.

Remark 54 (Data). *Particular (choose a total set), a large amount of, nonbiased, authentic (recoverable and with evidences) data are necessary and sufficient for our artificial intelligence. In general physics CA-pseudochaotic parameters also satisfy the reproducibility and treated by probability theory. It is necessary and sufficient for our artificial intelligence to be experienced to input it extreme data first and then more moderate ones (cf. necessary condition algorithm).*

9.2 Future prediction

Future prediction by artificial intelligence is, as a well-known evidence, as follows: Collect data by measuring things exactly (and use simulation if possible). Since there may exist CA-pseudochaotic parameters predict things probabilistically (by taking samples randomly) by artificial intelligence. Only near future CA-pseudochaotic parameters however may be predicted so that risk calculation is used. Note that imposing seemingly further risk may be making the original risk small.

9.3 Theory of punishment

To make risk small the following theory of punishment is needed: Our cost is the difficulty of solving the problem perfectly or approximately. Thus the punishment needs to be the cost that may be predicted because of accidents, damages or diseases in some areas and that is needed to solve the problem. In particular problem solving of mathematics, theory of measurement and future prediction are needed.

9.4 Discrimination

Definition 55. *Discrimination is a wrong definition and its wrong conclusions in the sense of our rubric.*

Theorem 56. *Solving discrimination is necessary for right discoveries in the sense of our rubric.*

Proof. By definition solving discrimination is necessary for realizing our rubric. □

Remark 57. *Our definition of discrimination has wrong exams in the sense of our rubric as a well-known evidences.*

Performance area	3	2	1
Importance of the troubles/discoveries	derive a solution with which the system becomes very stable for various natural states.	derive a solution with which the system becomes stable for some natural states.	derive a solution with which the system becomes unstable for various natural states.
Learning attitude	much experience of troubles. knowing the method of detection and problem solving and solving procedure exactly.	some experience of troubles. knowing the method of detection and problem solving and solving procedure.	little experience of troubles. not knowing the method of detection and problem solving and solving procedure.
Proof search	solving the problem with well-known conditions is almost impossible.	solving the problem with well-known conditions is difficult.	solving the problem with well-known conditions is easy.
Define problem	define problem clearly and insightfully with evidences for almost all relevant contextual factors.	define problem with evidences for some relevant contextual factors.	define problem not insightfully with no evidences.
Propose good solutions	propose a virtually perfect solution sensitive to the contextual, ethical, logical and cultural dimensions with consideration of history, feasibility and impacts and clearly show it is better than others.	propose a good solution, which is shown to be better than others.	propose no solution or show no goodness of the solution
Implement solution	with deep and thorough consideration of natural contextual factors.	with some consideration of natural contextual factors.	with little consideration of natural contextual factors.
Evaluate outcomes	review results relative to the problem defined with deep and thorough consideration of need for further work.	review results relative to the problem defined with some consideration of need for further work.	review results superficially with no consideration of need for further work.

表 1: Rubric of Problem Solving

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